

IMPROVING COSMOLOGICAL DISTANCE MEASUREMENTS BY RECONSTRUCTION OF THE BARYON ACOUSTIC PEAK

DANIEL J. EISENSTEIN¹, HEE-JONG SEO¹, EDWIN SIRKO², AND DAVID N. SPERGEL²

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ABSTRACT

The baryon acoustic oscillations are a promising route to the precision measure of the cosmological distance scale and hence the measurement of the time evolution of dark energy. We show that the non-linear degradation of the acoustic signature in the correlations of low-redshift galaxies is a correctable process. By suitable reconstruction of the linear density field, one can sharpen the acoustic peak in the correlation function or, equivalently, restore the higher harmonics of the oscillations in the power spectrum. With this, one can achieve better measurements of the acoustic scale for a given survey volume. Reconstruction is particularly effective at low redshift, where the non-linearities are worse but where the dark energy density is highest. At $z = 0.3$, we find that one can reduce the sample variance error bar on the acoustic scale by at least a factor of 2 and in principle by nearly a factor of 4. We discuss the significant implications our results have for the design of galaxy surveys aimed at measuring the distance scale through the acoustic peak.

Subject headings: large-scale structure of the universe — distance scale — cosmological parameters — cosmic microwave background

1. INTRODUCTION

The late-time acceleration of the expansion rate of the Universe (Riess et al. 1998; Perlmutter et al. 1999; Riess et al. 2004) argues for a remarkable change to our understanding of the forces of nature. Choosing between the exotic explanations to this surprising phenomenon may be possible through precision measure of the expansion rate of the Universe and of the growth of cosmological structure over time. Sound waves propagating in the first 400,000 years after the Big Bang produce a characteristic length scale in the anisotropies of the microwave background and in the clustering of galaxies (Peebles & Yu 1970; Sunyaev & Zel'dovich 1970; Bond & Efstathiou 1987; Holtzmann 1989; Hu & Sugiyama 1996; Eisenstein & Hu 1998). CMB experiments have clearly detected these fluctuations in both the temperature and polarization power spectrum (for a summary of recent results, see Spergel et al. 2006). With large galaxy surveys, we can detect this acoustic signature as a peak in the correlation function at ~ 150 Mpc or as a harmonic sequence of oscillations in the power spectrum (Cole et al. 2005; Eisenstein et al. 2005). This length scale can be measured as a characteristic angle on the sky, yielding the angular diameter distance as a function of redshift, and as a characteristic difference in redshift of galaxy pairs along the line of sight, yielding the Hubble parameter $H(z)$ (Eisenstein 2003; Blake & Glazebrook 2003; Hu & Haiman 2003; Seo & Eisenstein 2003). Because of the very large scale, the acoustic signature remains in the linear regime even today and is therefore a highly robust method for measuring the cosmological distance scale (Meiksin et al. 1999; Seo & Eisenstein 2005; Eisenstein et al. 2006).

However, as the universe evolves, the acoustic signature in the correlation function is broadened by non-linear effects. Non-linear gravitational structure formation and redshift distortions move galaxies away from their original locations, blurring out the peak at 150 Mpc separation, or, equivalently, erasing the higher harmonics in the power spectrum that represent smaller scales (Eisenstein et al. 2006). Even though the baryon bump remains *measurable* in the correlation function, these nonlinear effects reduce the *precision* of the distance scale that can be measured from a given volume of space, by roughly a factor of three at the present day. This loss is significant because survey volume is precious: larger surveys are more expensive and the amount of volume that we *can* survey in the low-redshift Universe is limited.

We argue in this paper that this loss of precision is avoidable. The blurring of the acoustic peak is largely due to bulk flows and super-cluster formation, effects that are generated by gravitational forces on large scales. The same map of galaxies intended to measure the acoustic scale is an accurate map of the large-scale gravitational source terms. One can essentially run the gravitational flow backwards to restore the acoustic peak nearly to its linear regime shape.

2. NON-LINEARITIES

Numerical simulations show that the advancing scale of non-linear gravitational collapse erases the higher harmonics of the acoustic oscillations (Meiksin et al. 1999; Seo & Eisenstein 2005; Springel et al. 2005; White 2005). Eisenstein et al. (2005, 2006) note that the harmonic sequence in the power spectrum correspond to a single peak in the correlation function and that the damping envelope corresponds to the broadening of this peak. In the configuration-space view, the source of the late-time broadening is clear: matter is being moved by roughly 10 Mpc from its initial position. Eisenstein et al. (2006) build a model for the non-linearity in terms of the differential motion of pairs initially separated by 150 Mpc.

¹ Steward Observatory, University of Arizona, 933 N. Cherry Ave., Tucson, AZ 85121

² Dept. of Astrophysical Sciences, Peyton Hall, Princeton University, Princeton, NJ 08544-1001

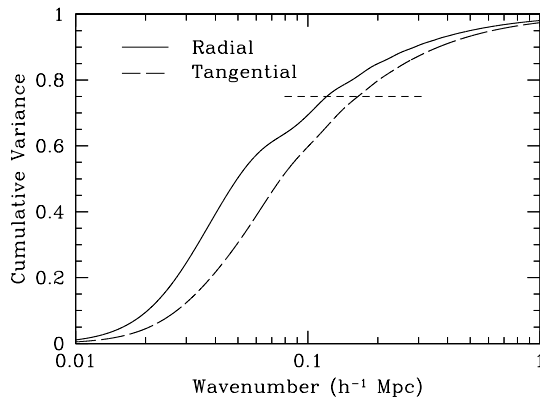


FIG. 1.— The cumulative variance in the differential motion of pairs initially separated by 150 Mpc as a function of cutoff wavenumber, normalized to the total variance. These curves are calculated in the Zel’dovich approximation according to the formulae in Eisenstein et al. (2006). The displacement along the separation vector is shown as the solid line; the displacement along a single direction perpendicular to the separation vector is shown as the dashed line. Most of the integral is supported by wavenumbers between 0.02 and $0.2h \text{ Mpc}^{-1}$. The horizontal dashed line is drawn at 75%, where half of the rms displacement has been fixed.

The final large-scale correlation function is simply the convolution of the linear correlation function with the distribution of differential motions.

One can ask what scales are responsible for the differential motion. Using the Zel’dovich approximation (Zeldovich 1970), the second moment of the distribution of the differential motion can be written as an integral over the power spectrum (Eisenstein et al. 2006). The cumulant of the integrand is plotted in Figure 1. Here we see that most of the motion is generated at $k \approx 0.1h \text{ Mpc}^{-1}$. Wavenumbers smaller than $0.02h \text{ Mpc}^{-1}$ contribute little, because these perturbations affect both points equally. Wavenumbers larger than $0.2h \text{ Mpc}^{-1}$ contribute less because the CDM power spectrum is fairly red on these scales.

The large-scale velocity field is responsible for most of the nonlinear effects that appear to “erase” the high order acoustic peaks in the power spectrum. The motions are dominantly due to bulk flows and the formation of superclusters. Small-scale formation of halos is subdominant; even big halos only pull material from an average of $5(M/10^{14}h^{-1} \text{ M}_\odot)^{1/3}h^{-1} \text{ Mpc}$ away, about half the bulk flow motions.

3. RECONSTRUCTION

Our major point is that because the scales of interest are large, the motions of galaxies can be well modeled by perturbation theory and in principle can be measured and removed. In the simplest terms, the bulk flows are generated by exactly the density perturbations that are being surveyed to measure the acoustic oscillations. The connection between the density and velocity field on these large scales is nearly that of linear theory, so one can predict the velocity field and undo the motion of the galaxies.

In more detail, reconstruction of the velocity field or the linear density field is a subject with considerable history. Peebles (1989, 1990) pioneered the subject by reconstructing the trajectories of Local Group

galaxies using the principle of least action, with the goal of constraining the local value of H_0 . The simple “Gaussianization” method of Weinberg (1992) approached the problem from a cosmological perspective. Nusser & Dekel (1992) and Gramann (1993) reconstructed primordial density fields with a technique based on the success of the Zeldovich approximation in the quasilinear regime, similar to our technique described below. Narayanan & Croft (1999) developed these techniques further (and offered a comparison of several different methods). Monaco & Efstathiou (1999) investigated a self-consistent, but more complicated, iterative scheme in which the initial densities source the Lagrangian map, rather than the final densities. Pursuit of this reconstruction method further may warrant using higher order relationships between peculiar velocities and densities (e.g., Chodorowski et al. 1998). The techniques of Croft & Gaztanaga (1997) and Brenier et al. (2003) are based on the principle of least action again, with the approximation that particles move on straight trajectories. As far as we know no one has attempted a full cosmologically relevant proper least action integration backwards in time, as the computational resources would be prohibitively expensive. Reversing the sign of gravity and running an evolved simulation backwards would not work, at least in the realistic case in which there are errors or unconstrained dimensions in the galaxies’ phase space, because the decaying modes would overpower the growing modes as one went back in time.

Restoring in full the acoustic signature at $k < 0.2h \text{ Mpc}^{-1}$ is an undemanding application of these reconstruction techniques. To demonstrate that reconstruction can help, we here show a simple method. We take the $z = 0.3$ outputs from the $512 h^{-1} \text{ Mpc}$ box N-body simulations from Seo & Eisenstein (2005) ($N = 256^3$ particles), compute the density field, Fourier transform and filter with a Gaussian of $10h^{-1} \text{ Mpc}$ or $20h^{-1} \text{ Mpc}$ width. From this, we predict the linear theory motion using $\nabla \cdot \vec{q} = -\delta$, where \vec{q} is the Lagrangian displacement field and δ is the fractional overdensity. We then move the particles by $-\vec{q}$. We do the same for a reference grid of smoothly distributed particles. In redshift space, we account for the linear redshift distortions (Kaiser 1987) by boosting the displacements along the line of sight by an additional factor of $1 + f$, where $f = d(\ln D)/d(\ln a)$, D is the linear growth function, and a is the scale factor of the Universe. A new density field is defined by the difference of the density field of the real particles and that of the reference particles. Note that, in contrast to the work of Sirko & Spergel (2006), the point is not to move all of the particles back to their initial location, but rather to move the measured densities back to their initial location.

The power spectrum of this field is shown in Figure 2. This Figure is the average of power spectra from 30 simulations at $z = 0.3$. One can see the degradation of the higher harmonics at $z = 0.3$ in the uncorrected density field, compared to the initial spectrum at $z = 49$, and see that the reconstruction has partially restored them. Figures 3 and 4 show the correlation functions in real and redshift space. The acoustic peak is nearly fully restored in real space. In redshift space, the peak is considerably improved but not fully fixed.

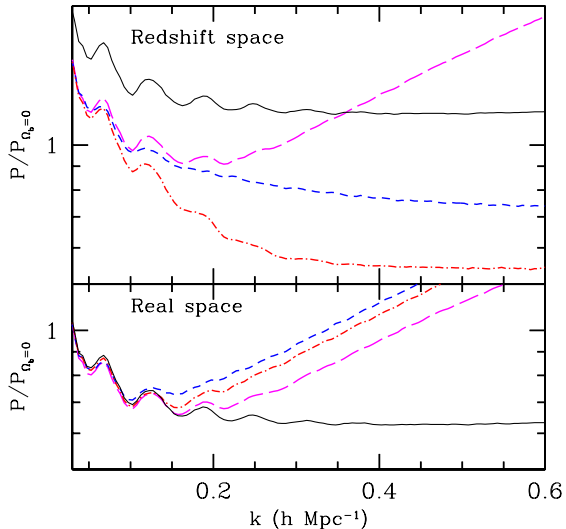


FIG. 2.— The matter power spectrum after reconstruction by the linear-theory density-velocity relation, with the density field Gaussian filtered. The bottom panel shows the real-space power spectrum; the top panel shows the spherically averaged redshift-space power spectrum. The black solid line shows the input power spectrum at $z = 49$; it has been displaced in the top panel for clarity. The blue short-dashed line shows the matter power spectrum at $z = 0.3$; one can see that acoustic peaks have been lost. In the bottom panel, the red dot-dashed line and magenta long-dashed line show the effects of reconstruction for $20h^{-1}$ Mpc and $10h^{-1}$ Mpc Gaussian filtering, respectively. In the top panel, both lines show $10h^{-1}$ Mpc filtering; the red dot-dashed line is without finger of God compression, while the magenta long-dashed line includes compression. The increase of power at large wavenumbers is essentially irrelevant to the quality of the acoustic signature; one would in practice marginalize over these broadband changes.

When one predicts the large-scale displacement field, one is also predicting the large-scale velocity field and hence the correction for large-scale peculiar velocities in redshift space. This is important because Eisenstein et al. (2006) find that redshift space distortions degrade the radial measurement of the acoustic peak. On large scales, the real-space displacements of particles are in the same direction as their peculiar velocity distortion, so the degradation of the acoustic peak is worse in redshift space than in real space. Reconstruction can fix this.

However, there are also redshift-space distortions from small-scale peculiar velocities, i.e., fingers of God. Clusters of galaxies appear as long cigars along the line of sight in redshift space. For the purposes of determining bulk flows, one should simply compress these fingers of God back to some approximation of their real-space location. Without this step, the fingers of God get stretched out further by the reconstruction, degrading the acoustic signature. To show that finger-of-God compression can help the reconstruction, we have identified clusters in redshift space with an anisotropic friends-of-friends algorithm and moved all cluster particles to the center of mass of the cluster. The correlation function that results from running our reconstruction on the compressed density field is shown in Figure 4. One sees a modest but useful improvement.

The simple reconstruction described above has not fully restored the linear acoustic scale, particularly when beginning from redshift space. We expect that more so-

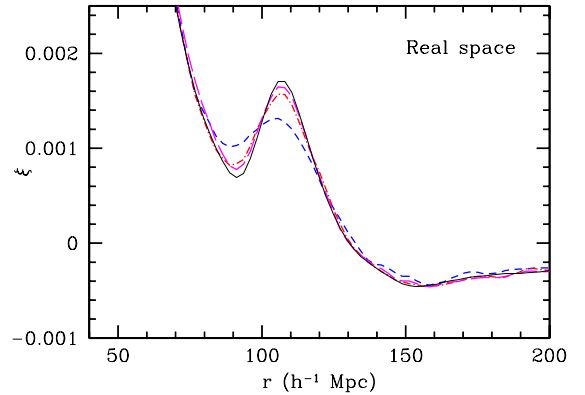


FIG. 3.— The real-space matter correlation function after reconstruction by the linear-theory density-velocity relation, with the density field Gaussian filtered. The black solid line shows the correlation function at $z = 49$. The blue short-dashed line shows it at $z = 0.3$; the acoustic peak has been smeared out. The red dot-dashed and magenta long-dashed lines show the effects of reconstruction for $20h^{-1}$ Mpc and $10h^{-1}$ Mpc Gaussian filtering, respectively. Even this very simple reconstruction recovers nearly all of the linear acoustic peak.

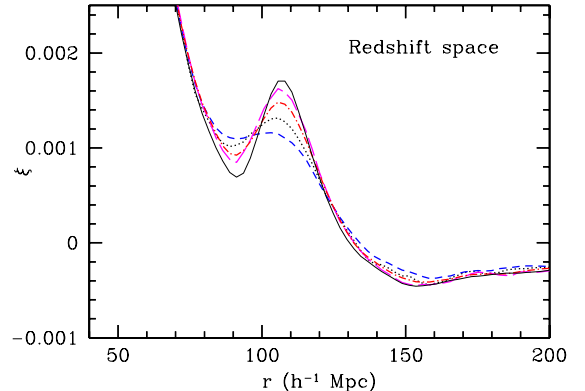


FIG. 4.— The redshift-space matter correlation function after reconstruction by the linear-theory density-velocity relation, with the density field Gaussian filtered. The black solid line shows the correlation function at $z = 49$. The blue short-dashed line shows the redshift-space correlation function at $z = 0.3$; the acoustic peak has been smeared out. The black dotted line shows the real-space correlation function for comparison. The red dot-dashed line shows the effects of reconstruction for a $10h^{-1}$ Mpc Gaussian filtering; the magenta long-dashed line is the result when one compresses the fingers of God prior to the reconstruction. These reconstructions significantly improve the acoustic peak.

phisticated reconstruction methods will produce further improvements. The small end of the range of the scales of interest are in the quasi-linear regime, and our assumption of linear theory for both the continuity equation and the redshift distortions is only a first approximation here.

We use a Fisher matrix calculation to estimate how much the reconstruction has improved the recovery of the acoustic scale. Our calculation is based on the methods in Seo & Eisenstein (2003) but with the derivatives multiplied by a Gaussian filtering that is tuned to match the pair-wise Lagrangian displacement (Eisenstein et al. 2006) and that visually reproduces the smearing of the acoustic peak. We focus here on the spherically-averaged acoustic scale; we will present anisotropic results (i.e., separate estimates for the angular diameter distance and the Hubble parameter) in a future paper. We find that

at $z = 0.3$, in the absence of reconstruction, a survey of $1h^{-3}\text{Gpc}^3$ should produce a distance measurement of 1.4% from real space and 1.9% from redshift space. The real-space value is in good agreement from the jack-knife estimates from the simulated real-space power spectra in (Seo & Eisenstein 2005). The simple reconstruction presented in this section improves these measurements to 0.75% and 0.95% in real and redshift space, respectively. This is a factor of two improvement. We find that if the reconstruction were perfect, one could achieve 0.5% distances (although shot noise from reasonable galaxy samples would degrade this to 0.55–0.60%). Hence, the raw precision at $z = 0.3$ is roughly a factor of 3.5 worse than the true cosmic variance, and the simplest reconstruction returns the first factor of two. At higher redshifts, the cosmic variance per unit volume won't change, but the data sets with or without reconstruction will be closer to that ideal.

4. DISCUSSION

We have demonstrated that density-field reconstruction can restore the linear-regime contrast of the baryon acoustic signature. Physically, the galaxy map locates the superclusters and voids and permits one to predict the large-scale flows resulting from these objects. By moving objects back to their initial location, one can correct the effect that these flows have on the characteristic separation of galaxies produced by the acoustic waves in the early universe. By restoring the linear-regime clustering, one can improve the precision of the measurement of the acoustic scale from galaxy redshift surveys and hence the available constraints on the cosmic distance scale and dark energy.

The opportunity for density reconstruction alters the optimization of surveys for baryon acoustic oscillations. Low-redshift surveys were thought to suffer in performance per unit volume; with reconstruction, this is not the case. Given that dark energy is more important (and hence easier to measure) at lower redshift, this will tend to push the optimal redshift range lower. On the other hand, lower redshift surveys were generally designed at lower target densities, because the suppression of the higher harmonics meant that it was not necessary to measure $k \approx 0.2h\text{Mpc}^{-1}$ well. Now the opportunity exists to get acoustic oscillation information from the higher harmonics, so that one might want to reduce shot noise and measure $k \approx 0.2h\text{Mpc}^{-1}$ well. In practice, with multi-object spectrographs, the number density at the highest redshifts will drop due to the flux limits imposed by a given exposure time, but at lower redshifts one can use the extra galaxies available to that flux limit, if one has sufficient number of fibers or slits.

Do the redshift surveys aimed at acoustic oscillations have enough information to do density-field reconstruction? Acoustic oscillation surveys are designed to balance shot noise and sample variance at the wavenumbers where the acoustic oscillations are found, namely $0.1 - 0.2h\text{Mpc}^{-1}$. If a survey is measuring at $k \lesssim 0.2h\text{Mpc}^{-1}$ with shot noise below sample variance, then it is producing a reasonable fidelity map at exactly the wavenumbers required to do the reconstruction, as shown in Figure 1. Turning this around, if the map is insufficient for reconstruction, it will also be too noisy to

measure the power spectrum well (i.e., approaching the sample variance limit) at the higher harmonics that one was hoping to improve. Indeed, within the approximation of the linear-theory density-displacement relation, the desired weighting of each Fourier mode to be used in reconstruction is $nP/(1 + nP)$, where n is the comoving number density and P is the power spectrum at that wavenumber. This is the same familiar factor that determines the measurement of modes in the power spectrum (Feldman, Kaiser, & Peacock 1994; Tegmark 1997). Hence, surveys designed to measure the acoustic oscillations with moderate signal-to-noise ratio maps will be well suited to reconstruction.

One must remember that the reconstruction need not be perfect. The propagation of sound waves at the epoch of recombination has mild dispersion that gives the linear regime acoustic peak a width of about 30 Mpc FWHM or $8h^{-1}\text{Mpc}$ rms width, comparable to the pairwise displacements of galaxies even at low redshifts (Eisenstein et al. 2006). Once we reduce the errors below this characteristic width, there is little improvement in the acoustic oscillations measurements. Thus, reconstruction is easier at higher redshifts: the raw displacements are smaller, so one can accomplish sufficient reconstruction using only larger scales. Even at $z = 3$, it is likely that reconstruction will benefit the recovery of the line-of-sight acoustic peak (and hence $H(z)$), because Eisenstein et al. (2006) shows that the redshift-space displacements are nearly double the real-space ones at $z \gtrsim 1$ and hence are not quite negligible even at $z = 3$.

Reconstruction places a premium on surveys with contiguous area. Surveys with lots of gaps on 10-100 Mpc scales will not measure the density field well enough to do these non-linear corrections: the source of bulk flows could be hidden in the gaps. Some gaps, e.g., due to bright stars, are inevitable and will degrade the reconstruction. Holes of 1 Mpc and smaller are less important; these are not biasing the density field at $k = 0.1h\text{Mpc}^{-1}$ much. Of course, the reconstruction techniques themselves must be able to deal with this mildly gappy data. Gaps are less of an issue at $z \gtrsim 2$ where reconstruction is not a large advantage.

Photometric redshift surveys for acoustic oscillations will not measure the 3-dimensional density field on the scales required to do density-field reconstruction. This is not a problem at $z > 2$, where the real-space density field is not degraded by non-linearities much anyways, but is a disadvantage relative to spectroscopic surveys at $z < 2$.

Much of the work on reconstruction has been focused on constructing the velocity field from the density field, or vice versa (Nusser et al. 1991). We are actually interested in constructing the displacement field from the density field. In linear theory, the velocity and displacement fields are the same, but differences enter at higher order in perturbation theory. We expect that much of the work on velocity-field reconstruction will be applicable to the displacement-field problem, and we note that in detail the displacement field may be somewhat easier to infer, as virialization affects the velocity field substantially but the displacement field very little.

The galaxies used in large-volume baryon acoustic oscillation surveys are typically biased, and it will be im-

portant to build reconstruction techniques that can handle this bias. Again, one must remember that only modest performance is required of the reconstruction; if one can fix half of the displacements, one will have achieved most of the gains. All that is required is that light trace mass reasonably well on large scales. This is well established by the simple behavior of bias (e.g. Tegmark et al. 2004), the detection of large-scale redshift distortions (e.g. Peacock et al. 2001), and the galaxy-mass cross-correlation measured from weak lensing (Sheldon et al. 2003). The errors that enter from imperfect knowledge of the underlying cosmology or exact level of bias (e.g., the value of the redshift distortion factor β) are much smaller than the required reconstruction precision.

Finally, while reconstruction can improve the precision of the acoustic scale measurement, one must also verify that the measurement remains unbiased. This will require further work, as one must also decide in detail how one is measuring the distance information from

the observed clustering, particularly in the case of an anisotropic clustering analysis. One would expect reconstruction to at least partially fix any bias that enters from non-linear gravity, but this has not yet been studied at the sub-percent level.

In conclusion, we believe that density-field reconstruction offers a significant opportunity to improve the baryon acoustic peak method by a factor of 2 to 3, particularly at lower redshifts.

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